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Congested, Excludable Public Goods
An Analytical Framework

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Abstract

This paper proposes a single-parameter characterization of the consumption technology for a congested public good where exclusion is possible and consumption levels may differ. The entire range of congestion possibilities (private to pure public goods) may be represented by varying the technological parameter. Analysis of optimal provision of congested, excludable public goods using the proposed framework suggests that public consumption levels should vary in the population to reflect demand differences. Uniform provision, which occurs with many public services, is generally suboptimal.

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by

Jan K. Brueckner

1. Introduction

Recognition of public good congestion has become widespread in the recent public finance literature. Congestion implies that a fixed amount of resources devoted to public production will generate less public consumption per capita the larger the size of the consuming group. The most common formalization of the congestion phenomenon assumes that per capita public consumption z is related to public output Q and group size n according to the expression $z = Qn^{-\gamma}$, where the parameter γ satisfies $0 \leq \gamma \leq 1$. In this formulation, the strength of congestion is inversely related to the degree of jointness-in-consumption which characterizes the good. When γ is set equal to zero, the above expression gives $z = Q$, which represents the case of a Samuelsonian (1954) pure public good where jointness-in-consumption is perfect and congestion is absent (z is independent of n). Setting $\gamma = 1$ yields $z = Q/n$, which corresponds to the case of a publicly-produced private good where jointness-in-consumption is non-existent (output is divided up among the consumers) and congestion is substantial (a 1% increase in n reduces z by 1%). When $0 < \gamma < 1$, jointness-in-consumption is intermediate between the extremes of pure public and private goods (output is neither divided up nor perfectly jointly consumed), and the strength of congestion is similarly

moderate. This intermediate case is no doubt relevant for many publicly-provided goods in the real world.[1]

A crucial feature of the above formulation is the implicit constraint that public consumption levels are identical for all members of the consuming group (z is the same for each of the n consumers). While this constraint will be appropriate for a public good such as national defense which has the property of nonexcludability, uniform consumption is not a technical necessity for other goods such as police and fire protection or education, where the possibility of exclusion means that consumption levels may differ across individuals. Clearly, the uniform-consumption constraint of the standard formulation will illegitimately oversimplify the public sector resource allocation problem when exclusion is possible. Indeed, since it will be socially optimal in general for consumption of congested, excludable goods to be non-uniform, efficiency analysis under a uniformity constraint necessarily carries a second-best interpretation.

The purpose of the present paper is to propose a specific framework for the analysis of congested, excludable public goods. The paper's principal innovation is the use of a one-parameter function which captures the congestion phenomenon without constraining public consumption levels to be uniform. The next section of the paper introduces this function and discusses its properties. The third section of the paper addresses the question of optimal provision of congested, excludable public goods using the proposed framework. The fourth section discusses conditions under which uniform provision of a congested, excludable public good may be socially desirable even in the presence of diverse tastes for public consumption. The final section contains conclusions.

2. The Analytical Framework

The issue of how public consumption is measured is important in any discussion of congested public goods. When $z = Qn^{-\gamma}$, it is clear that consumption and production are measured in the same units. In the case of police protection, for example, Q would equal the size of the police force while z would measure the number of police *effectively protecting* each individual. While use of identical units is somewhat unnatural (in the police case, response time might be a more appropriate measure of protection), this approach is clearly necessary in any framework which attempts to include private and pure public goods as limiting cases (units of measurement must be identical for such goods).[2] For this reason, the framework developed below also assumes identical units of measurement.

The most general representation of a public consumption technology where consumption levels may differ across individuals is a relationship of the form

$$Q = f(z_1, z_2, \ldots, z_n), \tag{1}$$

which gives the minimum production level Q needed to sustain a specified public consumption vector in a group of size n. Since consumption need not be uniform under (1), it is no longer possible to define congestion in terms of the effect of an increase in group size n on the uniform level of per capita consumption. Recalling, however, that the strength of congestion is simply the mirror image of the degree of jointness-in-consumption in the standard framework, it is natural to adopt a jointness-in-consumption view of congestion in the present context. Thus, the implied measure of congestion in the following discussion will be the degree of jointness-in-consumption exhibited by the technology (1) (a synonym for congestion in the present framework is consumption "rivalness").

The existence of jointness-in-consumption means that total consumption of the publicly-provided good is greater than production. In the standard formulation, jointness-in-consumption requires $Q < nz$, which follows when $\gamma < 1$ (note that $nz = Qn^{1-\gamma}$). In the present context, the existence of jointness-in-consumption means that (1) must satisfy

$$Q < \Sigma z_i. \tag{2}$$

Inequality (2) will not hold, of course, in the case of a publicly-produced private good. The absence of jointness-in-consumption for such a good means that production and total consumption are identically equal, so that (1) must reduce to $Q = \Sigma z_i$. On the other hand, when (1) represents an excludable pure public good, jointness-in-consumption is potentially perfect but consumption levels nevertheless may differ. In this case, (2) can be replaced by the more precise statement

$$Q = \max \{z_i\} \tag{3}$$

since output clearly must equal the largest consumption level in the group. It is important to realize that although (3) is predicated on non-uniform public consumption, it will never be socially optimal for $z_j < Q$ to hold for any j when the public good is pure (consumption may be increased costlessly for such an individual j). Specification of the consumption technology, however, must recognize the feasibility of non-uniform consumption even when such a pattern may not be optimal.

A one-parameter function with the general form of (1) which has the desired properties is

$$Q = (\Sigma z_i^{\theta})^{1/\theta}, \quad \theta \geq 1. \tag{4}$$

When $\theta = 1$, (4) reduces to the private good expression $Q = \Sigma z_i$. When $\theta > 1$ and $z_i > 0$ for at least two values of i, the jointness-in-consumption property $Q < \Sigma z_i$ follows from the standard inequality $\Sigma z_i^{\theta} < (\Sigma z_i)^{\theta}$. Finally, it is easy to see that

$$\lim_{\theta \to \infty} (\Sigma z_i^{\theta})^{1/\theta} = \max \{z_i\}, \tag{5}$$

so that as θ approaches infinity, the relationship (4) approaches (3), the one appropriate for a pure public good. The proof of (5) consists of rewriting the LHS of (4) as

$$z_{max} \, [\Sigma(z_i/z_{max})^{\theta}]^{1/\theta}, \tag{6}$$

where $z_{max} \equiv \max \{z_i\}$, and noting that $(z_i/z_{max})^{\theta} \to 0$ for $z_i \neq z_{max}$ as $\theta \to \infty$, so that the bracketed expression converges to unity when z_{max} is unique (to an integer between 1 and n otherwise) while its exponent converges to zero. Thus, the specification (4) has all the features appropriate to a consumption technology for a congested, excludable public good, with the type of good portrayed changing from a private to pure public good (and congestion decreasing correspondingly) as θ increases from one to infinity.[3] Finally, when the z_i are constrained to be equal, (4) reduces to $Q = (nz^{\theta})^{1/\theta}$ or

$$z = Qn^{-1/\theta}. \qquad (7)$$

This relationship is identical to the standard formulation since $1/\theta$ will range between one and zero as θ increases from one to infinity.[4]

Figure 1 shows graphs of "iso-output" contours (loci of z_i such that $(\Sigma z_i^{\theta})^{1/\theta} = \overline{Q}$) for the case n = 2. The straight line contour corresponds to the private good case, the quarter-circle contour represents the intermediate case $\theta = 2$, and the right-angled contour represents the pure public good case.

To understand the intuitive meaning of the specification (4), it is helpful to consider a concrete example. Suppose the public good in question is police protection, with Q measuring as before the size of the police force. The z_i again represent the numbers of police effectively protecting individual houses (against burglary, say). Since i's neighborhood may be patrolled more frequently than j's, z_i need not equal z_j; in this sense, exclusion is feasible. The public good is obviously congested since increasing the frequency of patrol in any neighborhood requires more police, other things equal (in other words, the pure public good case does not apply). However, the protection technology is characterized by jointness-in-consumption since the total number of police Q is less than the sum of the effective numbers of police protecting the various houses (a patrol car protects many houses simultaneously by its presence in a neighborhood). Note that if burglars are deterred by a relatively infrequent patrol, θ will be large, indicating substantial jointness-in-consumption; bolder burglars would lead to a lower θ and a situation closer to the private good case.

3. Optimal Provision of a Congested, Excludable Public Good

The analysis of optimality will proceed under the assumption that individuals consume a single private good (x_i) in addition to the public good. Letting $U_i(x_i,z_i)$ denote the utility function for individual i and λ_i denote i's welfare weight in a linear social welfare function, the Lagrangean expression for the welfare maximization problem is

$$\Sigma\lambda_i U_i(x_i,z_i) - \psi F(\Sigma x_i, (\Sigma z_i^{\theta})^{1/\theta}), \tag{8}$$

where F = 0 characterizes the production possibility curve for the economy.[6] The optimality conditions are

$$\frac{U_{k2}}{U_{k1}} = \frac{F_2}{F_1}\frac{\partial Q}{\partial z_k} = \frac{F_2}{F_1}\left(\frac{z_k^{\theta}}{\Sigma z_i^{\theta}}\right)^{1-\frac{1}{\theta}} \qquad k = 1, 2, \ldots, n. \tag{9}$$

When $\theta = 1$, the term multiplying F_2/F_1 becomes unity and (9) reduces to the familiar set of optimality conditions for private goods: $MRS_k = MRT$ must hold for all individuals k. When $\theta > 1$, however, it is easy to see that $(z_k^{\theta}/\Sigma z_i^{\theta})^{1-\frac{1}{\theta}} < 1$, so that $MRS_k < MRT$ holds for all k. The existence of jointness-in-consumption makes it suboptimal for individual MRS's to fully mirror the trade-off between private and public production.

In the case of a pure public good ($\theta = \infty$), solution of the welfare maximization problem is best accomplished in a two-step procedure which does not make use of (9). First, since non-maximal z_i values may be increased up to $\max\{z_i\}$ with no resource cost, the social optimum clearly requires equality among the z_i.[7] $\mathrm{Max}\{z_i\}$ then equals the common value z, which may be chosen optimally in standard fashion, yielding the usual condition $\Sigma MRS_i = MRT$ (the relationship $MRS_k < MRT$ continues to hold).[8]

While $\Sigma MRS_i = MRT$ holds for a pure public good, ΣMRS_i will exceed MRT when the public good is congested. In the private good case, $\Sigma MRS_i = nMRT$, and it is easily shown that the relationship

$$nMRT \geq \Sigma MRS_i \geq MRT \tag{10}$$

holds for arbitrary $\theta \geq 1$, with the strict inequalities applying when $1 < \theta < \infty$. The validity of the first inequality in (10) is established by summing both sides of (9) and noting that when $\theta > 1$, $\sum_k (z_k^\theta / \sum_i z_i^\theta)^{1-\frac{1}{\theta}} < n$ follows from the previously-noted fact that each term in the summation is less than one. Satisfaction of the second inequality in (10) requires that the above sum exceeds unity, which is equivalent to the requirement $(\Sigma z_i^{\theta-1})^{1/(\theta-1)} > (\Sigma z_i^\theta)^{1/\theta}$. The validity of the last inequality follows from the fact that $(\Sigma z_i^\alpha)^{1/\alpha}$ is a decreasing function of α.[9] Eq. (10) shows that the familiar $\Sigma MRS_i = MRT$ optimality rule is simply a limiting case in a general congested public goods framework.[10]

The feasibility of exclusion in the present framework means that consumption levels may be high for high demanders of the public good while resources may be saved by restricting the public consumption of low demanders. By ruling out such skewed consumption patterns when the public good is congested and excludable, welfare maximization under the standard uniform-consumption approach leads to a suboptimal outcome. To see how consumption is related to demand differences at the optimum under the present framework, consider a simple example where utility functions are $U_i(x_i,z_i) = x_i+h_i(z_i)$, with $h_i' > 0$, $h_i'' < 0$. Suppose further that the ranking of the h_i functions according to the value of h_i' is the same regardless of the magnitude of z, and let individuals be numbered in ascending order of h_i' ($h_i' < h_{i+1}'$, etc.). Since h_i' is simply the inverse demand function for the public good, the ranking of consumers reflects the strength of their public good demands. To see that high demanders enjoy higher public consumption at the optimum, note that rearrangement of the condition (9) implies that $h_i'(z_i)z_i^{1-\theta} = h_j'(z_j)z_j^{1-\theta}$ for $i \neq j$. Suppose $i > j$, implying that i is a higher demander than j, and suppose further that $z_j \geq z_i$. Since $\theta \geq 1$, $z_j^{1-\theta} \leq z_i^{1-\theta}$ holds, and since $h_j'(z) < h_i'(z)$, it follows that $h_j'(z_j) < h_i'(z_j) \leq h_i'(z_i)$, where the second inequality uses $h_i'' < 0$. Combining these results yields $h_i'(z_i)z_i^{1-\theta} > h_j'(z_j)z_j^{1-\theta}$, which contradicts the optimality condition and establishes the impossibility of $z_j \geq z_i$. Hence, public consumption is positively correlated with the strength of demand under the given assumptions. Returning to the police example of the previous section, this result translates into the natural proposition that social welfare maximization requires police protection to be skewed in favor of those citizens who, because of frailty or some other vulnerability, are especially fearful of burglary.

The analysis so far has assumed that the social planner knows the utility function of each consumer in the economy. This, of course, is an unreasonable assumption, and its relaxation raises the familiar problem of preference revelation. As long as the cost of public consumption is correlated with the stated strength of a consumer's demand, individuals may have an incentive to understate demand in pursuit of a large surplus. For example, if the government were to pursue benefit taxation, charging each consumer an individualized price per unit of consumption equal to the RHS of (9) evaluated at the solution based on stated preferences, high demanders may try to reduce their presumably high price per unit[11] by understating their preferences. It is important to realize that the preference revelation problem need not disappear when θ becomes small. Since any degree of jointness-in-consumption leads to different individualized prices for the public good (see (9)), an incentive for strategic behavior to reduce the cost of public consumption may exist even when θ is small and the consumption technology is close to that of a private good. In the private good case, however, individualized prices will be identical and equal to marginal production cost for the good, and since no individual cost advantage can be achieved by misstatement of preferences, truthful revelation will occur.

4. Potential Optimality of Uniform Public Consumption

Real-world consumption levels are in fact uniform within jurisdictional boundaries for many congested, excludable public goods (examples are police and fire protection and public education), an outcome which would appear to be suboptimal given the preceding discussion. While the difficulty of ascertaining individual public good demands (truthful or otherwise) may account for this institutional arrangement, a simple and realistic modification of the analysis suggests that uniform public consumption may be socially optimal even when true demands are known. The modification consists of recognizing that a diversity of public consumption levels may entail an administrative resource cost which would not arise under a simpler regime where consumption is uniform. Letting δ represent administrative cost in terms of the private good, which equals zero when the z_i are identical but assumes a positive value c otherwise, the economy's production constraint becomes $F(\Sigma x_i - \delta, (\Sigma z_i^{\theta})^{1/\theta}) = 0$.[12] Clearly, an allocation where consumption is uniform and $\delta = 0$ may be superior to one where the z_i differ and $\delta = c$. Thus, the standard representation of a congested public good ($z = Qn^{-\gamma}$) may be appropriate regardless of whether or not exclusion is possible provided that the administrative cost of sustaining consumption diversity is sufficiently high.

5. Conclusion

This paper has presented the first single-parameter characterization of a consumption technology for a congested, excludable public good. The framework permits relaxation of the uniform-consumption constraint implicit in the standard congested public goods model and allows individual public consumption levels to reflect demand differences, leading to a higher level of social welfare. Since most publicly-provided goods are excludable, normative analysis which aspires to real-world relevance clearly must incorporate a framework like the one presented in this paper.

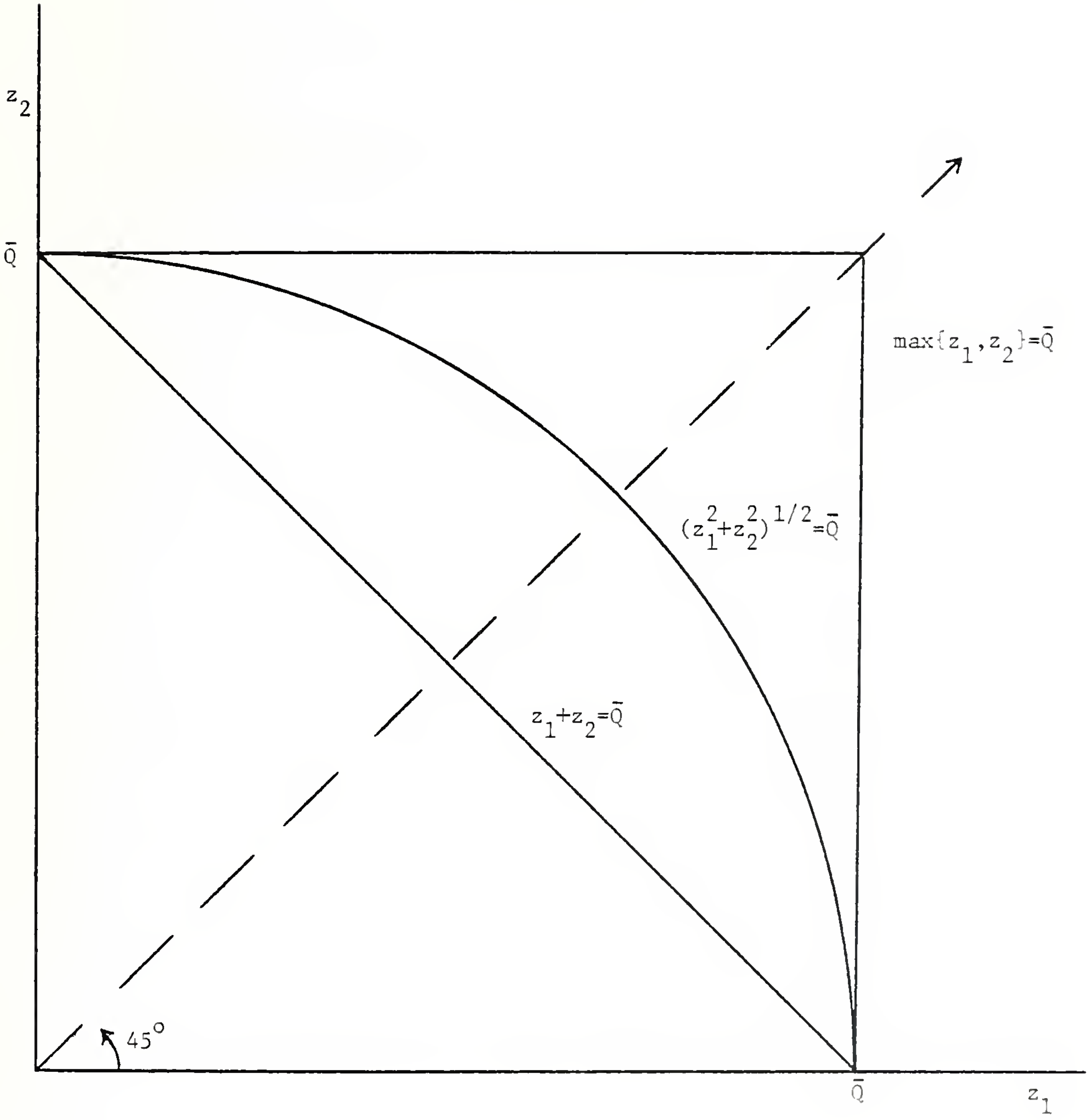

Figure 1.

Footnotes

*I wish to thank Jon Sonstelie, Lanny Arvan, and Francoise Schoumaker for comments. Any errors, however, are my own.

[1]Brueckner (1981) found that γ for fire protection services is approximately .25, indicating substantial jointness-in-consumption.

[2]See Buchanan (1968) for a discussion of the problem of units of measurement. Brueckner (1981) measured consumption and production in different units in the case of fire protection.

[3]The appropriateness of the formulation (4) was suggested by the properties of the commonly-used social welfare function $W = (\Sigma U_i^{\sigma})^{1/\sigma}$, where $\sigma \leq 1$. The Benthamite case $W = \Sigma U_i$ corresponds to $\sigma = 1$, while the Rawlsian case $W = \min\{U_i\}$ corresponds to $\sigma = -\infty$. See Atkinson (1970).

[4]At this point, the existence of two other approaches to public good congestion should be noted. Oakland (1972) assumes that an individual's public consumption may equal any amount up to a capacity level Q, with congestion entering the utility function via a term equal to the ratio of the sum of individual consumption levels and Q (this term has a negative partial derivative). Sandmo (1973) takes the view that benefits from a public good are generated by joint consumption with private goods. Congestion results from excessive use of these private "inputs" (too many cars on a freeway, for example).

[5]Stiglitz and Atkinson (1980), Ch. 16, present a diagram showing the private and pure public good contours of Figure 1. They did not, however, propose a way of handling intermediate cases.

[6]The marginal conditions for the maximization of a linear social welfare function also characterize a Pareto-efficient allocation.

[7]If the marginal utility of public consumption falls to zero for some individual i, there is, of course, no benefit from increasing z_i. Since doing so is costless, however, as long as z_i is non-maximal, the conclusion in the text still holds.

[8]In an example where it is possible to compute a closed-form solution to the optimality conditions, the convergence of the solution to the pure public good outcome as $\theta \to \infty$ may be directly verified. The example assumes that utility functions are $z_i + x_i^{\beta_i}$ and that the

transformation function is $\Sigma x_i + (\Sigma z_i^{\theta})^{1/\theta} = B$, where B is some positive constant. The solution for public consumption levels gives

$$z_i = (\lambda_i/\Omega)^{\frac{1}{\theta-1}} \left[B - \Sigma \left(\frac{\lambda_i \beta_i}{\Omega} \right)^{\frac{1}{1-\beta_i}} \right] ,$$

where $\Omega = \left(\Sigma \lambda_i^{\frac{\theta}{\theta-1}} \right)^{\frac{\theta-1}{\theta}}$. As $\theta \to \infty$, the different z_i solutions converge to $B - \Sigma(\frac{\lambda_i \beta_i}{\Sigma \lambda_i})^{\frac{1}{1-\beta_i}}$, which equals the z level satisfying $\Sigma MRS_i = MRT$.

[9]Straightforward calculations show that

$$\frac{d}{d\alpha}(\Sigma z_i^{\alpha})^{1/\alpha} = \frac{1}{\alpha^2}(\Sigma z_i^{\alpha})^{\frac{1}{\alpha}-1}(\Sigma z_i^{\alpha}[\log z_i^{\alpha} - \log(\Sigma_j z_j^{\alpha})]),$$

which is less than zero.

[10]The relationship (10) will, of course, hold when public consumption is constrained to be uniform.

[11]Recalling that in the preceding example, public consumption at the optimum was positively correlated with the strength of demand, it is clear from (9) that individualized prices will be similarly correlated with demand. Note that the individualized price which leads a given consumer to demand his optimal z is a function of all the optimal consumption levels, including his own.

[12]An alternative formulation might assume a fixed administrative cost plus a variable cost positively related to the variance of public consumption.

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